



OWL 2 Web Ontology Language Direct Semantics

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Abstract

The OWL 2 Web Ontology Language, informally OWL 2, is an ontology language for the Semantic Web with formally defined meaning. OWL 2 ontologies provide classes, properties, individuals, and data values and are stored as Semantic Web documents. OWL 2 ontologies can be used along with information written in RDF, and OWL 2 ontologies themselves are primarily exchanged as RDF documents. The OWL 2 [Document Overview](#) describes the overall state of OWL 2, and should be read before other OWL 2 documents.

This document provides the direct model-theoretic semantics for OWL 2, which is compatible with the description logic *SROIQ*. Furthermore, this document defines the most common inference problems for OWL 2.

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Summary of Changes

This document has undergone only minor editorial changes since the previous version of 21st April, 2009.

Please Comment By 30 July 2009

The [OWL Working Group](#) seeks to gather experience from [implementations](#) in order to increase confidence in the language and meet specific [exit criteria](#). This document will remain a Candidate Recommendation until at least 30 July 2009. After that date, when and if the exit criteria are met, the group intends to request [Proposed Recommendation](#) status.

Please send reports of implementation experience, and other feedback, to public-owl-comments@w3.org ([public archive](#)). Reports of any success or difficulty with the [test cases](#) are encouraged. Open discussion among developers is welcome at public-owl-dev@w3.org ([public archive](#)).

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1 Introduction

This document defines the direct model-theoretic semantics of OWL 2. The semantics given here is strongly related to the semantics of description logics [[Description Logics](#)] and it extends the semantics of the description logic *SROIQ* [[SROIQ](#)]. As the definition of *SROIQ* does not provide for datatypes and punning, the semantics of OWL 2 is defined directly on the constructs of the structural specification of OWL 2 [[OWL 2 Specification](#)] instead of by reference to *SROIQ*. For the constructs available in *SROIQ*, the semantics of *SROIQ* trivially corresponds to the one defined in this document.

Since each OWL 1 DL ontology is an OWL 2 ontology, this document also provides a direct semantics for OWL 1 Lite and OWL 1 DL ontologies; this semantics is equivalent to the direct model-theoretic semantics of OWL 1 Lite and OWL 1 DL [[OWL 1 Semantics and Abstract Syntax](#)]. Furthermore, this document also provides the direct model-theoretic semantics for the OWL 2 profiles [[OWL 2 Profiles](#)].

The semantics is defined for OWL 2 axioms and ontologies, which should be understood as instances of the structural specification [[OWL 2 Specification](#)]. Parts of the structural specification are written in this document using the functional-style syntax.

OWL 2 allows ontologies, anonymous individuals, and axioms to be annotated; furthermore, annotations themselves can contain additional annotations. All these types of annotations, however, have no semantic meaning in OWL 2 and are ignored in this document. OWL 2 declarations are used only to disambiguate class expressions from data ranges and object property from data property expressions in the functional-style syntax; therefore, they are not mentioned explicitly in this document.

2 Direct Model-Theoretic Semantics for OWL 2

This section specifies the direct model-theoretic semantics of OWL 2 ontologies.

2.1 Vocabulary

A *datatype map*, formalizing [datatype maps](#) from the OWL 2 Specification [[OWL 2 Specification](#)], is a 6-tuple $D = (N_{DT}, N_{LS}, N_{FS}, \cdot^{DT}, \cdot^{LS}, \cdot^{FS})$ with the following components:

- N_{DT} is a set of datatypes (more precisely, names of datatypes) that does not contain the datatype *rdfs:Literal*.
- N_{LS} is a function that assigns to each datatype $DT \in N_{DT}$ a set $N_{LS}(DT)$ of strings called *lexical forms*. The set $N_{LS}(DT)$ is called the *lexical space* of DT .
- N_{FS} is a function that assigns to each datatype $DT \in N_{DT}$ a set $N_{FS}(DT)$ of pairs (F, v) , where F is a *constraining facet* and v is an arbitrary data value called the *constraining value*. The set $N_{FS}(DT)$ is called the *facet space* of DT .
- For each datatype $DT \in N_{DT}$, the *interpretation function* \cdot^{DT} assigns to DT a set $(DT)^{DT}$ called the *value space* of DT .
- For each datatype $DT \in N_{DT}$ and each lexical form $LV \in N_{LS}(DT)$, the *interpretation function* \cdot^{LS} assigns to the pair (LV, DT) a *data value* $(LV, DT)^{LS} \in (DT)^{DT}$.
- For each datatype $DT \in N_{DT}$ and each pair $(F, v) \in N_{FS}(DT)$, the *interpretation function* \cdot^{FS} assigns to (F, v) the set $(F, v)^{FS} \subseteq (DT)^{DT}$.

A *vocabulary* $V = (V_C, V_{OP}, V_{DP}, V_I, V_{DT}, V_{LT}, V_{FA})$ over a datatype map D is a 7-tuple consisting of the following elements:

- V_C is a set of [classes](#) as defined in the OWL 2 Specification [[OWL 2 Specification](#)], containing at least the classes *owl:Thing* and *owl:Nothing*.
- V_{OP} is a set of [object properties](#) as defined in the OWL 2 Specification [[OWL 2 Specification](#)], containing at least the object properties *owl:topObjectProperty* and *owl:bottomObjectProperty*.

- V_{DP} is a set of *data properties* as defined in the OWL 2 Specification [OWL 2 Specification], containing at least the data properties *owl:topDataProperty* and *owl:bottomDataProperty*.
- V_I is a set of *individuals* (named and anonymous) as defined in the OWL 2 Specification [OWL 2 Specification].
- V_{DT} is a set containing all datatypes of D , the datatype *rdfs:Literal*, and possibly other datatypes; that is, $N_{DT} \cup \{ rdfs:Literal \} \subseteq V_{DT}$.
- V_{LT} is a set of *literals* LV^{DT} for each datatype $DT \in N_{DT}$ and each lexical form $LV \in N_{LS}(DT)$.
- V_{FA} is the set of pairs (F, It) for each constraining facet F , datatype $DT \in N_{DT}$, and literal $It \in V_{LT}$ such that $(F, (LV, DT_1)^{LS}) \in N_{FS}(DT)$, where LV is the lexical form of It and DT_1 is the datatype of It .

Given a vocabulary V , the following conventions are used in this document to denote different syntactic parts of OWL 2 ontologies:

- OP denotes an object property;
- OP_E denotes an object property expression;
- DP denotes a data property;
- DP_E denotes a data property expression;
- C denotes a class;
- CE denotes a class expression;
- DT denotes a datatype;
- DR denotes a data range;
- a denotes an individual (named or anonymous);
- lt denotes a literal; and
- F denotes a constraining facet.

2.2 Interpretations

Given a datatype map D and a vocabulary V over D , an *interpretation* $I = (\Delta_I, \Delta_D, \cdot^C, \cdot^{OP}, \cdot^{DP}, \cdot^I, \cdot^{DT}, \cdot^{LT}, \cdot^{FA})$ for D and V is a 9-tuple with the following structure:

- Δ_I is a nonempty set called the *object domain*.
- Δ_D is a nonempty set disjoint with Δ_I called the *data domain* such that $(DT)^{DT} \subseteq \Delta_D$ for each datatype $DT \in V_{DT}$.
- \cdot^C is the *class interpretation function* that assigns to each class $C \in V_C$ a subset $(C)^C \subseteq \Delta_I$ such that
 - $(owl:Thing)^C = \Delta_I$ and
 - $(owl:Nothing)^C = \emptyset$.
- \cdot^{OP} is the *object property interpretation function* that assigns to each object property $OP \in V_{OP}$ a subset $(OP)^{OP} \subseteq \Delta_I \times \Delta_I$ such that
 - $(owl:topObjectProperty)^{OP} = \Delta_I \times \Delta_I$ and
 - $(owl:bottomObjectProperty)^{OP} = \emptyset$.
- \cdot^{DP} is the *data property interpretation function* that assigns to each data property $DP \in V_{DP}$ a subset $(DP)^{DP} \subseteq \Delta_I \times \Delta_D$ such that
 - $(owl:topDataProperty)^{DP} = \Delta_I \times \Delta_D$ and
 - $(owl:bottomDataProperty)^{DP} = \emptyset$.

- \cdot^I is the *individual interpretation function* that assigns to each individual $a \in V_I$ an element $(a)^I \in \Delta_I$.
- \cdot^{DT} is the *datatype interpretation function* that assigns to each datatype $DT \in V_{DT}$ a subset $(DT)^{DT} \subseteq \Delta_D$ such that
 - \cdot^{DT} is the same as in D for each datatype $DT \in N_{DT}$, and
 - $(rdfs:Literal)^{DT} = \Delta_D$.
- \cdot^{LT} is the *literal interpretation function* that is defined as $(It)^{LT} = (LV, DT)^{LS}$ for each $It \in V_{LT}$, where LV is the lexical form of It and DT is the datatype of It .
- \cdot^{FA} is the *facet interpretation function* that is defined as $(F, It)^{FA} = (F, (It)^{LT})^{FS}$ for each $(F, It) \in V_{FA}$.

The following sections define the extensions of \cdot^{OP} , \cdot^{DT} , and \cdot^C to object property expressions, data ranges, and class expressions.

2.2.1 Object Property Expressions

The object property interpretation function \cdot^{OP} is extended to object property expressions as shown in Table 1.

Table 1. Interpreting Object Property Expressions

Object Property Expression	Interpretation \cdot^{OP}
ObjectInverseOf(OP)	$\{(x, y) \mid (y, x) \in (OP)^{OP}\}$

2.2.2 Data Ranges

The datatype interpretation function \cdot^{DT} is extended to data ranges as shown in Table 3. All datatypes in OWL 2 are unary, so each datatype DT is interpreted as a unary relation over Δ_D — that is, as a set $(DT)^{DT} \subseteq \Delta_D$. OWL 2 currently does not define data ranges of arity more than one; however, by allowing for n -ary data ranges, the syntax of OWL 2 provides a "hook" allowing implementations to introduce extensions such as comparisons and arithmetic. An n -ary data range DR is interpreted as an n -ary relation $(DR)^{DT}$ over Δ_D — that is, as a set $(DR)^{DT} \subseteq (\Delta_D)^n$.

Table 3. Interpreting Data Ranges

Data Range	Interpretation \cdot^{DT}
DataIntersectionOf(DR ₁ ... DR _n)	$(DR_1)^{DT} \cap \dots \cap (DR_n)^{DT}$
DataUnionOf(DR ₁ ... DR _n)	$(DR_1)^{DT} \cup \dots \cup (DR_n)^{DT}$
DataComplementOf(DR)	$(\Delta_D)^n \setminus (DR)^{DT}$ where n is the arity of DR
DataOneOf(lt ₁ ... lt _n)	$\{(lt_1)^{LT}, \dots, (lt_n)^{LT}\}$

DatatypeRestriction(DT F ₁ lt ₁ ... F _n lt _n)	$(DT)^{DT} \cap (F_1, lt_1)^{FA} \cap \dots \cap (F_n, lt_n)^{FA}$
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2.2.3 Class Expressions

The class interpretation function \cdot^C is extended to class expressions as shown in Table 4. For S a set, #S denotes the number of elements in S.

Table 4. Interpreting Class Expressions

Class Expression	Interpretation \cdot^C
ObjectIntersectionOf(CE ₁ ... CE _n)	$(CE_1)^C \cap \dots \cap (CE_n)^C$
ObjectUnionOf(CE ₁ ... CE _n)	$(CE_1)^C \cup \dots \cup (CE_n)^C$
ObjectComplementOf(CE)	$\Delta_I \setminus (CE)^C$
ObjectOneOf(a ₁ ... a _n)	$\{(a_1)^I, \dots, (a_n)^I\}$
ObjectSomeValuesFrom(OPE CE)	$\{x \mid \exists y : (x, y) \in (OPE)^{OP} \text{ and } y \in (CE)^C\}$
ObjectAllValuesFrom(OPE CE)	$\{x \mid \forall y : (x, y) \in (OPE)^{OP} \text{ implies } y \in (CE)^C\}$
ObjectHasValue(OPE a)	$\{x \mid (x, (a)^I) \in (OPE)^{OP}\}$
ObjectHasSelf(OPE)	$\{x \mid (x, x) \in (OPE)^{OP}\}$
ObjectMinCardinality(n OPE)	$\{x \mid \#\{y \mid (x, y) \in (OPE)^{OP}\} \geq n\}$
ObjectMaxCardinality(n OPE)	$\{x \mid \#\{y \mid (x, y) \in (OPE)^{OP}\} \leq n\}$
ObjectExactCardinality(n OPE)	$\{x \mid \#\{y \mid (x, y) \in (OPE)^{OP}\} = n\}$
ObjectMinCardinality(n OPE CE)	$\{x \mid \#\{y \mid (x, y) \in (OPE)^{OP} \text{ and } y \in (CE)^C\} \geq n\}$
ObjectMaxCardinality(n OPE CE)	$\{x \mid \#\{y \mid (x, y) \in (OPE)^{OP} \text{ and } y \in (CE)^C\} \leq n\}$

ObjectExactCardinality(n OPE CE)	$\{ x \mid \#\{ y \mid (x, y) \in (OPE)^{OP} \text{ and } y \in (CE)^C \} = n \}$
DataSomeValuesFrom(DPE ₁ ... DPE _n DR)	$\{ x \mid \exists y_1, \dots, y_n : (x, y_k) \in (DPE_k)^{DP} \text{ for each } 1 \leq k \leq n \text{ and } (y_1, \dots, y_n) \in (DR)^{DT} \}$
DataAllValuesFrom(DPE ₁ ... DPE _n DR)	$\{ x \mid \forall y_1, \dots, y_n : (x, y_k) \in (DPE_k)^{DP} \text{ for each } 1 \leq k \leq n \text{ imply } (y_1, \dots, y_n) \in (DR)^{DT} \}$
DataHasValue(DPE lt)	$\{ x \mid (x, (lt)^{LT}) \in (DPE)^{DP} \}$
DataMinCardinality(n DPE)	$\{ x \mid \#\{ y \mid (x, y) \in (DPE)^{DP} \} \geq n \}$
DataMaxCardinality(n DPE)	$\{ x \mid \#\{ y \mid (x, y) \in (DPE)^{DP} \} \leq n \}$
DataExactCardinality(n DPE)	$\{ x \mid \#\{ y \mid (x, y) \in (DPE)^{DP} \} = n \}$
DataMinCardinality(n DPE DR)	$\{ x \mid \#\{ y \mid (x, y) \in (DPE)^{DP} \text{ and } y \in (DR)^{DT} \} \geq n \}$
DataMaxCardinality(n DPE DR)	$\{ x \mid \#\{ y \mid (x, y) \in (DPE)^{DP} \text{ and } y \in (DR)^{DT} \} \leq n \}$
DataExactCardinality(n DPE DR)	$\{ x \mid \#\{ y \mid (x, y) \in (DPE)^{DP} \text{ and } y \in (DR)^{DT} \} = n \}$

2.3 Satisfaction in an Interpretation

An interpretation $I = (\Delta_I, \Delta_D, \cdot^C, \cdot^{OP}, \cdot^{DP}, \cdot^I, \cdot^{DT}, \cdot^{LT}, \cdot^{FA})$ satisfies an axiom w.r.t. an ontology O if the axiom satisfies the relevant condition from the following sections. Satisfaction of axioms in I is defined w.r.t. O because satisfaction of key axioms uses the following function:

$ISNAMED_O(x) = true$ for $x \in \Delta_I$ if and only if $(a)^I = x$ for some named individual a occurring in the [axiom closure](#) of O

2.3.1 Class Expression Axioms

Satisfaction of OWL 2 class expression axioms in I w.r.t. O is defined as shown in Table 5.

Table 5. Satisfaction of Class Expression Axioms in an Interpretation

Axiom	Condition
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SubClassOf(CE ₁ CE ₂)	$(CE_1)^C \subseteq (CE_2)^C$
EquivalentClasses(CE ₁ ... CE _n)	$(CE_j)^C = (CE_k)^C$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$
DisjointClasses(CE ₁ ... CE _n)	$(CE_j)^C \cap (CE_k)^C = \emptyset$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$
DisjointUnion(C CE ₁ ... CE _n)	$(C)^C = (CE_1)^C \cup \dots \cup (CE_n)^C$ and $(CE_j)^C \cap (CE_k)^C = \emptyset$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$

2.3.2 Object Property Expression Axioms

Satisfaction of OWL 2 object property expression axioms in I w.r.t. O is defined as shown in Table 6.

Table 6. Satisfaction of Object Property Expression Axioms in an Interpretation

Axiom	Condition
SubObjectPropertyOf(OPE ₁ OPE ₂)	$(OPE_1)^{OP} \subseteq (OPE_2)^{OP}$
SubObjectPropertyOf(ObjectPropertyChain(OPE ₁ ... OPE _n) OPE)	$\forall y_0, \dots, y_n : (y_0, y_1) \in (OPE_1)^{OP}$ and ... and $(y_{n-1}, y_n) \in (OPE_n)^{OP}$ imply $(y_0, y_n) \in (OPE)^{OP}$
EquivalentObjectProperties(OPE ₁ ... OPE _n)	$(OPE_j)^{OP} = (OPE_k)^{OP}$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$
DisjointObjectProperties(OPE ₁ ... OPE _n)	$(OPE_j)^{OP} \cap (OPE_k)^{OP} = \emptyset$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$
ObjectPropertyDomain(OPE CE)	$\forall x, y : (x, y) \in (OPE)^{OP}$ implies $x \in (CE)^C$
ObjectPropertyRange(OPE CE)	$\forall x, y : (x, y) \in (OPE)^{OP}$ implies $y \in (CE)^C$
InverseObjectProperties(OPE ₁ OPE ₂)	$(OPE_1)^{OP} = \{ (x, y) \mid (y, x) \in (OPE_2)^{OP} \}$
FunctionalObjectProperty(OPE)	$\forall x, y_1, y_2 : (x, y_1) \in (OPE)^{OP}$ and $(x, y_2) \in (OPE)^{OP}$ imply $y_1 = y_2$

InverseFunctionalObjectProperty(OPE)	$\forall x_1, x_2, y : (x_1, y) \in (OPE)^{OP}$ and $(x_2, y) \in (OPE)^{OP}$ imply $x_1 = x_2$
ReflexiveObjectProperty(OPE)	$\forall x : x \in \Delta_I$ implies $(x, x) \in (OPE)^{OP}$
IrreflexiveObjectProperty(OPE)	$\forall x : x \in \Delta_I$ implies $(x, x) \notin (OPE)^{OP}$
SymmetricObjectProperty(OPE)	$\forall x, y : (x, y) \in (OPE)^{OP}$ implies $(y, x) \in (OPE)^{OP}$
AsymmetricObjectProperty(OPE)	$\forall x, y : (x, y) \in (OPE)^{OP}$ implies $(y, x) \notin (OPE)^{OP}$
TransitiveObjectProperty(OPE)	$\forall x, y, z : (x, y) \in (OPE)^{OP}$ and $(y, z) \in (OPE)^{OP}$ imply $(x, z) \in (OPE)^{OP}$

2.3.3 Data Property Expression Axioms

Satisfaction of OWL 2 data property expression axioms in I w.r.t. O is defined as shown in Table 7.

Table 7. Satisfaction of Data Property Expression Axioms in an Interpretation

Axiom	Condition
SubDataPropertyOf(DPE ₁ DPE ₂)	$(DPE_1)^{DP} \subseteq (DPE_2)^{DP}$
EquivalentDataProperties(DPE ₁ ... DPE _n)	$(DPE_j)^{DP} = (DPE_k)^{DP}$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$
DisjointDataProperties(DPE ₁ ... DPE _n)	$(DPE_j)^{DP} \cap (DPE_k)^{DP} = \emptyset$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$
DataPropertyDomain(DPE CE)	$\forall x, y : (x, y) \in (DPE)^{DP}$ implies $x \in (CE)^C$
DataPropertyRange(DPE DR)	$\forall x, y : (x, y) \in (DPE)^{DP}$ implies $y \in (DR)^{DT}$
FunctionalDataProperty(DPE)	$\forall x, y_1, y_2 : (x, y_1) \in (DPE)^{DP}$ and $(x, y_2) \in (DPE)^{DP}$ imply $y_1 = y_2$

2.3.4 Datatype Definitions

Satisfaction of datatype definitions in I w.r.t. O is defined as shown in Table 8.

Table 8. Satisfaction of Datatype Definitions in an Interpretation

Axiom	Condition
DatatypeDefinition(DT DR)	$(DT)^{DT} = (DR)^{DT}$

2.3.5 Keys

Satisfaction of keys in I w.r.t. O is defined as shown in Table 9.

Table 9. Satisfaction of Keys in an Interpretation

Axiom	Condition
HasKey(CE (OPE ₁ ... OPE _m) (DPE ₁ ... DPE _n))	$\forall x, y, z_1, \dots, z_m, w_1, \dots, w_n :$ if $x \in (CE)^C$ and $ISNAMED_O(x)$ and $y \in (CE)^C$ and $ISNAMED_O(y)$ and $(x, z_i) \in (OPE_i)^{OP}$ and $(y, z_i) \in (OPE_i)^{OP}$ and $ISNAMED_O(z_i)$ for each $1 \leq i \leq m$ and $(x, w_j) \in (DPE_j)^{DP}$ and $(y, w_j) \in (DPE_j)^{DP}$ for each $1 \leq j \leq n$ then $x = y$

2.3.6 Assertions

Satisfaction of OWL 2 assertions in I w.r.t. O is defined as shown in Table 10.

Table 10. Satisfaction of Assertions in an Interpretation

Axiom	Condition
SameIndividual(a ₁ ... a _n)	$(a_j)^I = (a_k)^I$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$
DifferentIndividuals(a ₁ ... a _n)	$(a_j)^I \neq (a_k)^I$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$
ClassAssertion(CE a)	$(a)^I \in (CE)^C$
ObjectPropertyAssertion(OPE a ₁ a ₂)	$((a_1)^I, (a_2)^I) \in (OPE)^{OP}$

NegativeObjectPropertyAssertion(OPE a ₁ a ₂)	$((a_1)^I, (a_2)^I) \notin (OPE)^{OP}$
DataPropertyAssertion(DPE a lt)	$((a)^I, (lt)^{LT}) \in (DPE)^{DP}$
NegativeDataPropertyAssertion(DPE a lt)	$((a)^I, (lt)^{LT}) \notin (DPE)^{DP}$

2.3.7 Ontologies

An interpretation I *satisfies* an OWL 2 ontology O if all axioms in the [axiom closure](#) of O (with anonymous individuals standardized apart as described in Section 5.6.2 of the OWL 2 Specification [[OWL 2 Specification](#)]) are satisfied in I w.r.t. O .

2.4 Models

Given a datatype map D , an interpretation $I = (\Delta_I, \Delta_D, \cdot^C, \cdot^{OP}, \cdot^{DP}, \cdot^I, \cdot^{DT}, \cdot^{LT}, \cdot^{FA})$ for D is a *model* of an OWL 2 ontology O w.r.t. D if an interpretation $J = (\Delta_J, \Delta_D, \cdot^C, \cdot^{OP}, \cdot^{DP}, \cdot^J, \cdot^{DT}, \cdot^{LT}, \cdot^{FA})$ for D exists such that \cdot^J coincides with \cdot^I on all named individuals and J satisfies O .

Thus, an interpretation I satisfying O is also a model of O . In contrast, a model I of O may not satisfy O directly; however, by modifying the interpretation of anonymous individuals, I can always be coerced into an interpretation J that satisfies O .

2.5 Inference Problems

Let D be a datatype map and V a vocabulary over D . Furthermore, let O and O_1 be OWL 2 ontologies, CE , CE_1 , and CE_2 class expressions, and a a named individual, such that all of them refer only to the vocabulary elements in V . Furthermore, *variables* are symbols that are not contained in V . Finally, a *Boolean conjunctive query* Q is a closed formula of the form

$$\exists x_1, \dots, x_n, y_1, \dots, y_m : [A_1 \wedge \dots \wedge A_k]$$

where each A_i is an *atom* of the form $C(s)$, $OP(s, t)$, or $DP(s, u)$ with C a class, OP an object property, DP a data property, s and t individuals or some variable x_j , and u a literal or some variable y_j .

The following inference problems are often considered in practice.

Ontology Consistency: O is *consistent* (or *satisfiable*) w.r.t. D if a model of O w.r.t. D and V exists.

Ontology Entailment: O entails O_1 w.r.t. D if every model of O w.r.t. D and V is also a model of O_1 w.r.t. D and V .

Ontology Equivalence: O and O_1 are *equivalent* w.r.t. D if O entails O_1 w.r.t. D and O_1 entails O w.r.t. D .

Ontology Equisatisfiability: O and O_1 are *equisatisfiable* w.r.t. D if O is satisfiable w.r.t. D if and only if O_1 is satisfiable w.r.t. D .

Class Expression Satisfiability: CE is satisfiable w.r.t. O and D if a model $I = (\Delta_I, \Delta_D, \cdot^C, \cdot^{OP}, \cdot^{DP}, \cdot^I, \cdot^{DT}, \cdot^{LT}, \cdot^{FA})$ of O w.r.t. D and V exists such that $(CE)^C \neq \emptyset$.

Class Expression Subsumption: CE_1 is *subsumed* by a class expression CE_2 w.r.t. O and D if $(CE_1)^C \subseteq (CE_2)^C$ for each model $I = (\Delta_I, \Delta_D, \cdot^C, \cdot^{OP}, \cdot^{DP}, \cdot^I, \cdot^{DT}, \cdot^{LT}, \cdot^{FA})$ of O w.r.t. D and V .

Instance Checking: a is an *instance* of CE w.r.t. O and D if $(a)^I \in (CE)^C$ for each model $I = (\Delta_I, \Delta_D, \cdot^C, \cdot^{OP}, \cdot^{DP}, \cdot^I, \cdot^{DT}, \cdot^{LT}, \cdot^{FA})$ of O w.r.t. D and V .

Boolean Conjunctive Query Answering: Q is an *answer* w.r.t. O and D if Q is true in each model of O w.r.t. D and V according to the standard definitions of first-order logic.

In order to ensure that ontology entailment, class expression satisfiability, class expression subsumption, and instance checking are decidable, the following restriction w.r.t. O needs to be satisfied:

Each class expression of type **MinObjectCardinality**, **MaxObjectCardinality**, **ExactObjectCardinality**, and **ObjectHasSelf** that occurs in O_1 , CE , CE_1 , and CE_2 can contain only object property expressions that are [simple](#) in the [axiom closure](#) Ax of O .

For ontology equivalence to be decidable, O_1 needs to satisfy this restriction w.r.t. O and vice versa. These restrictions are analogous to the first condition from Section 11.2 of the OWL 2 Specification [[OWL 2 Specification](#)].

3 Independence of the Direct Semantics from the Datatype Map in OWL 2 DL (Informative)

OWL 2 DL has been defined so that the consequences of an OWL 2 DL ontology O do not depend on the choice of a datatype map, as long as the datatype map chosen contains all the datatypes occurring in O . This statement is made precise by the following theorem, and it has several useful consequences:

- One can apply the direct semantics to an OWL 2 DL ontology O by considering only the datatypes explicitly occurring in O .

- When referring to various reasoning problems, the datatype map D need not be given explicitly, as it is sufficient to consider an implicit datatype map containing only the datatypes from the given ontology.
- OWL 2 DL reasoners can provide datatypes not explicitly mentioned in this specification without fear that this will change the meaning of OWL 2 DL ontologies not using these datatypes.

Theorem DS1. Let O_1 and O_2 be OWL 2 DL ontologies over a vocabulary V and $D = (N_{DT}, N_{LS}, N_{FS}, \cdot^{DT}, \cdot^{LS}, \cdot^{FS})$ a datatype map such that each datatype mentioned in O_1 and O_2 is *rdfs:Literal*, a datatype defined in the respective ontology, or it occurs in N_{DT} . Furthermore, let $D' = (N_{DT'}, N_{LS'}, N_{FS'}, \cdot^{DT'}, \cdot^{LS'}, \cdot^{FS'})$ be a datatype map such that $N_{DT} \subseteq N_{DT'}$, $N_{LS}(DT) = N_{LS'}(DT)$, and $N_{FS}(DT) = N_{FS'}(DT)$ for each $DT \in N_{DT}$, and $\cdot^{DT'}$, $\cdot^{LS'}$, and $\cdot^{FS'}$ are extensions of \cdot^{DT} , \cdot^{LS} , and \cdot^{FS} , respectively. Then, O_1 entails O_2 w.r.t. D if and only if O_1 entails O_2 w.r.t. D' .

Proof. Without loss of generality, one can assume O_1 and O_2 to be in negation-normal form [Description Logics]. Furthermore, since datatype definitions in O_1 and O_2 are acyclic, one can assume that each defined datatype has been recursively replaced with its definition; thus, all datatypes in O_1 and O_2 are from $N_{DT} \cup \{rdfs:Literal\}$. The claim of the theorem is equivalent to the following statement: an interpretation I w.r.t. D and V exists such that O_1 is and O_2 is not satisfied in I if and only if an interpretation I' w.r.t. D' and V exists such that O_1 is and O_2 is not satisfied in I' . The (\Leftarrow) direction is trivial since each interpretation I w.r.t. D' and V is also an interpretation w.r.t. D and V . For the (\Rightarrow) direction, assume that an interpretation $I = (\Delta_I, \Delta_D, \cdot^C, \cdot^{OP}, \cdot^{DP}, \cdot^I, \cdot^{DT}, \cdot^{LT}, \cdot^{FA})$ w.r.t. D and V exists such that O_1 is and O_2 is not satisfied in I . Let $I' = (\Delta_I, \Delta_{D'}, \cdot^{C'}, \cdot^{OP'}, \cdot^{DP'}, \cdot^I, \cdot^{DT'}, \cdot^{LT'}, \cdot^{FA'})$ be an interpretation such that

- $\Delta_{D'}$ is obtained by extending Δ_D with the value space of all datatypes in $N_{DT'} \setminus N_{DT}$,
- $\cdot^{C'}$ coincides with \cdot^C on all classes, and
- $\cdot^{DP'}$ coincides with \cdot^{DP} on all data properties apart from *owl:topDataProperty*.

Clearly, $DataComplementOf(DR)^{DT} \subseteq DataComplementOf(DR)^{DT'}$ for each data range DR that is either a datatype, a datatype restriction, or an enumerated data range. The *owl:topDataProperty* property can occur in O_1 and O_2 only in tautologies. The interpretation of all other data properties is the same in I and I' , so $(CE)^C = (CE)^{C'}$ for each class expression CE occurring in O_1 and O_2 . Therefore, O_1 is and O_2 is not satisfied in I' . QED

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5 References

5.1 Normative References

[OWL 2 Specification]

[OWL 2 Web Ontology Language: Structural Specification and Functional-Style Syntax](#) Boris Motik, Peter F. Patel-Schneider, Bijan Parsia, eds. W3C Candidate Recommendation, 11 June 2009, <http://www.w3.org/TR/2009/CR-owl2-syntax-20090611/>. Latest version available at <http://www.w3.org/TR/owl2-syntax/>.

5.2 Nonnormative References

[Description Logics]

[The Description Logic Handbook: Theory, Implementation, and Applications, second edition](#). Franz Baader, Diego Calvanese, Deborah McGuinness, Daniele Nardi, and Peter Patel-Schneider, eds. Cambridge University Press, 2007. Also see the [Description Logics Home Page](#).

[OWL 1 Semantics and Abstract Syntax]

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[OWL 2 Profiles]

[OWL 2 Web Ontology Language: Profiles](#) Boris Motik, Bernardo Cuenca Grau, Ian Horrocks, Zhe Wu, Achille Fokoue, Carsten Lutz, eds. W3C Candidate Recommendation, 11 June 2009, <http://www.w3.org/TR/2009/CR-owl2-profiles-20090611/>. Latest version available at <http://www.w3.org/TR/owl2-profiles/>.

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